Stage 5: Mathematics STEM Advanced Pathway

Quick Sketch sample program

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<th>Overview</th>
<th>Duration</th>
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<td>In Quick Sketch students will recall, consolidate and develop the following essential skills:</td>
<td>5 weeks</td>
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<td>▪ all equations describe a shape: a line, a curve or a region.</td>
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<td>▪ changes to the equation cause changes to the shape, and that the nature of these changes is consistent across different types of graphs.</td>
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<td>▪ the transformations: reflection, dilation, translation using both the Cartesian plane and coordinate pairs.</td>
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<td>This unit will focus on:</td>
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<td>▪ sketching graphs from equations presented in fully factorised, fully expanded and partially factorised form.</td>
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<td>▪ the key characteristics of the graph presented by each form without having to algebraically manipulate between forms themselves.</td>
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<td>If this unit is presented before algebraic techniques for factorising it can provide a context for those processes when they are subsequently studied.</td>
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<td>In Quick Sketch students will develop the following essential STEM understandings:</td>
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<td>▪ that the shapes and movements of all things in the universe can be described with equations</td>
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<td>▪ that describing shape and movement allows people to communicate with precision about location and movement both with each other and with machinery.</td>
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Outcomes

A student:

▪ graphs simple non-linear relationships (MA5.1-7NA)
▪ connects algebraic and graphical representations of simple non-linear relationships (MA5.2-10NA)
▪ sketches and interprets a variety of non-linear relationships (MA5.3-9NA)
▪ solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations (MA5.3-7NA)
▪ uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures (MA5.3-1WM)
<table>
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<th>Common misconceptions</th>
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<td>Students may:</td>
<td>Curves and lines</td>
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<td>▪ still be confused about the ‘−’ symbol and hence have trouble interpreting negative</td>
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<td>coefficients</td>
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<td>▪ have an undeveloped understanding of zero</td>
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<td>▪ still be developing their understanding of equations as relationships between</td>
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<td>variables and hence the significance of coordinate pairs as locating points</td>
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<td>belonging to a curve</td>
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<td>▪ have undeveloped ‘drawing skills’ that affect their confidence when sketching a</td>
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<td>smooth curve</td>
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<td>▪ be resistant to sketching and want to plot points from a table of values for every</td>
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<td>graph – indicating that they have not yet recognised that key features of graphs</td>
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<td>are sufficient to describe a curve or line</td>
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<td>▪ not be ready to accept ‘undefined’ and/or ‘no solutions’ as valid answers</td>
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| • graph simple non-linear relations, with and without the use of digital technologies (ACMNA296) | **Activity:** Students play ‘catch’:
  1. Outdoors if possible but a screwed up piece of paper could be used in the classroom. The ‘ball’ is passed randomly the class can try for a record run of catches without the ball being dropped. Then, discuss how people can throw and catch without too much deliberate thought.
  2. With a (very) soft ball, students can try and get a ball through a hoop while blindfolded.
    a. First when they are allowed an initial look at the hoop before shooting blindfolded.
    b. Secondly, blindfolded throughout with a sighted ‘instructor’ to guide them with spoken instructions.
   Students analyse the decisions and calculations the brain is making in all three activities.
   **Link to learning:** Students are usually unaware of the complex calculations they do ‘without thinking’. Catching a ball requires the mind to predict the parabolic path that the ball will take, based on observations of the force and direction of the throw and compensating for environmental factors such as wind or odd ball shape. Then the brain works out how to ‘get its body’ into the best position for the catch, all the while adjusting for any changes in the ball’s path and coordinating the whole body to position hands and absorb the impact of the ball. During a | • Video: Robots today: view the following videos:
  • **Video:** ‘Hot Robot at SXSW says she wants to destroy humans’:
    https://www.youtube.com/watch?v=W0_DPioPmF0&feature=youtu.be
  • **Video:** ‘Are robots today in competition with surgeons?’:
    https://www.youtube.com/watch?v=MobYs34z5x4&feature=youtu.be
  • **STEM:** Watch the video: ‘Projectile Motion & Parabolas – Science of NFL Football’:
    https://www.youtube.com/watch?v=HB4ws7R0A3M&feature=youtu.be which shows the motion of the ball is shown as a parabola. Also shows the velocity vectors. Pause the video at intervals to allow students to absorb information.
  | • **STEM:** Depending on school resources, students can be developing their understanding of how equations describe motion in preparation for:
    • programming a robot to move
    • programming a turtle to move in an online setting
    • programming an object to move in a created game world
    • using equations to create an artwork. |
<p>| • identify parabolic shapes in the environment (Reasoning) | | |
| • use digital technologies to investigate and describe features of the graphs of parabolas given in the following forms for both positive and negative values of a and k, eg $y = ax^2, y = ax^2 + c, y = (x + h)^2, y = (x + h)^2 + k$ (Communicating, Reasoning) | | |
| • graph parabolic relationships of the form $y = ax^2, y = ax^2 + c$, with and without the use of digital technologies | | |
| • describe the effect on the graph of $y = x^2$ of multiplying $x^2$ by different numbers (including negative numbers) or of adding different numbers (including negative numbers) to $x^2$ (Communicating, Reasoning) | | |
| • determine the equation of a parabola, given a graph of the parabola with the main features clearly indicated (Reasoning) | | |</p>
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| competitive game the brain also builds in calculations based on the movement of every other player in the game. For a robot to catch a ball, every conscious and unconscious thought that a person has must be converted into an equation. Students watch the video: ‘3D monocular robotic ball catching’: [https://www.researchgate.net/publication/259135336_3D_monocular_robotic_ball_catching](https://www.researchgate.net/publication/259135336_3D_monocular_robotic_ball_catching) for an impression of this. **Consolidation for skill development:** The teacher explicitly teaches, with an emphasis on vocabulary:  
  - the ‘shape’ of a ball’s flight through the air is a parabola – sketch a diagram (do not plot points)  
  - a demonstration (outdoors if possible, or indoors with a bean bag substitute)  
  - ‘Shooting hoops’ – a netball hoop is better as there is no backboard.  
  - ‘Keepings off’ – passing between pairs over the head of those in between them.  
  - students to note the ‘flight path’ of every throw.  
  - students to ‘shoot hoops’ in a way that the ball does not trace out a parabola.  
  - that the equation for this shape is a quadratic  
    
    \[ y = x^2 \]  
  - what can be noticed about the equation. |
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<td>conclusions that changes to the basic equation will change the shape of the basic graph – the changes follow a consistent set of rules</td>
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<td>Reflections (upright or upside down)</td>
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<td>Dilatations (wider or narrower)</td>
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<td>Translations (shifted up, down, left or right)</td>
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<td><em>(Note to teachers: do not explain more at this stage; allow students to discover for themselves in ‘Investigate the Parabola’)</em></td>
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<td><strong>Investigate:</strong> Students use the following worksheets:</td>
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<td>• ‘Investigate the Parabola 2’: <a href="https://www.tes.com/teaching-resource/investigate-the-parabola-2-key-features-11339530">https://www.tes.com/teaching-resource/investigate-the-parabola-2-key-features-11339530</a></td>
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<td>• ‘Investigate the Parabola 3’: <a href="https://www.tes.com/teaching-resource/investigate-the-parabola-3-how-the-shape-is-formed-spreadsheet-activity-11437469">https://www.tes.com/teaching-resource/investigate-the-parabola-3-how-the-shape-is-formed-spreadsheet-activity-11437469</a></td>
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<td><strong>Consolidation for skill development:</strong></td>
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<td>The teacher directs the writing of summary notes with worked examples drawing together the content met in the three investigations. Include vertex, maximum and minimum as</td>
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<td>alternatives for ‘turning point’. The teacher explicitly teaches (with all examples in turning point form):</td>
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<td>▪ calculating the $y$-intercept by substituting $x = 0$ into the equation</td>
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<td>▪ calculating the $x$-intercept by substituting $y = 0$ into the equation and noting the two solutions to $x^2 = p$ and including examples where $p$ is negative and hence no solution exists so no $x$-intercepts exist</td>
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<td><strong>Guided practice:</strong> School-based and online worksheets could be used as resources, such as:</td>
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<td>▪ ‘Graphing parabolas in vertex form’: <a href="http://www.augustatech.edu/math/molik/GraphParabVertForm.pdf">http://www.augustatech.edu/math/molik/GraphParabVertForm.pdf</a></td>
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<td>▪ ‘Graph quadratics in vertex form’: <a href="https://www.khanacademy.org/math/algebra/quadratics/vertex-form-alg1/e/graphing_parabolas_1">https://www.khanacademy.org/math/algebra/quadratics/vertex-form-alg1/e/graphing_parabolas_1</a></td>
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<td>▪ find $y$-intercepts for the graph of $y = ax^2 + bx + c$, given $a$, $b$ and $c$</td>
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<td>▪ find $x$-intercepts, where appropriate, for the</td>
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<td><strong>Introduction:</strong> The teacher explicitly teaches:</td>
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<td>▪ that a quadratic equation has a number of representations and extension of the</td>
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<td><strong>STEM:</strong> Watch slow motion videos of parabolas in the real world, such as ‘Snowboard slow motion montage’: <a href="https://www.youtube.com/watch?v=OiXaXRWv">https://www.youtube.com/watch?v=OiXaXRWv</a></td>
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<td>graph of $y = (x - h)(x - k)$</td>
<td>- determine the equation of the axis of symmetry of a parabola using the midpoint of the interval joining the points at which the parabola cuts the $x$-axis.</td>
<td><a href="https://www.youtube.com/watch?v=qhM&amp;feature=youtu.be">qhM&amp;feature=youtu.be</a> noting the shape of ramps and flight.</td>
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<td>- factorising and expanding skills from Stage 4 will allow you to change from one form to another. (That will be studied in the next topic.) For now, you can focus on understanding what each representation can tell you without too much effort on your part.</td>
<td>(If resources allow, this unit could include an excursion to a skate-park or excursion to an exhibition event where ramps could be observed setting 'riders' up for parabolic 'flights'. Equally, the class could observe a ball-sport match and pay particular attention to the way players moved the ball towards goal or over nets.)</td>
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<td>- that they have already you have seen turning point form and found that it immediately shows the parabola’s orientation and location of the vertex, and that fairly simple calculations give the $x$ and $y$ intercepts. This makes it a very useful form of the quadratic.</td>
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<td>- that a fully expanded form of the quadratic is most useful for showing you the $y$-intercept is $y = ax^2 + bx + c$</td>
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<td>- that a fully factorised form of the quadratic is most useful for showing you the $x$-intercepts is $y = (x - h)(x - k)$</td>
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<td>- that the equation of the axis of symmetry of a parabola using the midpoint of the interval joining the points at which the parabola cuts the $x$-axis can be found.</td>
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<td>Consolidation for skill development:</td>
<td>The teacher explicitly teaches:</td>
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<td>- examples of equations and their graphs.</td>
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<td>- that the $y$-intercept should be found by substituting $x = 0$ into the equation – because at the $y$-intercept the $x$-coordinate is zero</td>
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<td>- to name and justify the Null Factor Law:</td>
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<td>that the $x$-intercept to be found by substituting $y = 0$ – because at the $x$-intercept the $y$-coordinate is zero</td>
<td><strong>Introduction:</strong> students watch the video: ‘Red Bull X-Fighters World Tour 2012 Sydney’: [<a href="https://www.youtube.com/watch?v=caXHOTvii">https://www.youtube.com/watch?v=caXHOTvii</a> mU&amp;feature=youtu.be](<a href="https://www.youtube.com/watch?v=caXHOTvii">https://www.youtube.com/watch?v=caXHOTvii</a> mU&amp;feature=youtu.be) The teacher challenges students to work out the equations describing this rider’s path based on estimated distances and heights. Students complete the worksheet: ‘Mathematical Dirt Bike Challenge’: <a href="https://www.tes.com/teaching-resource/mathematical-dirt-bike-challenge-11408407">https://www.tes.com/teaching-resource/mathematical-dirt-bike-challenge-11408407</a> Part 1 challenges students to find the quadratic equation that describes the bike’s ‘flight’. Part 2 leads students into learning about the equation of a circle. <strong>Link to learning:</strong> <strong>Engineering:</strong> Increase student awareness of the breadth and depth of Engineering-related careers. From the designers of the bikes to the bulldozer drivers sculpting the dirt banks, this video is loaded with engineering in the real world. show the video: ‘Red Bull X-Fighters World Tour 2012 Sydney’: [<a href="https://www.youtube.com/watch?v=caXHOTvii">https://www.youtube.com/watch?v=caXHOTvii</a> mU&amp;feature=youtu.be](<a href="https://www.youtube.com/watch?v=caXHOTvii">https://www.youtube.com/watch?v=caXHOTvii</a> mU&amp;feature=youtu.be) which shows dirt bikes doing aerial stunts. Note the shape of the ramps and the flights. Note that two types of motion here – a parabolic flight path, and a circular rotation of the bike and rider. <strong>Technology:</strong> Circles in digital design. Allow students to ‘play with’ the applet: ‘SpinGraph’: <a href="http://www.scootle.edu.au/ec/viewing/L10092/h">http://www.scootle.edu.au/ec/viewing/L10092/h</a></td>
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<td>that the axis of symmetry as a vertical line with an equation</td>
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<td>(Note to teachers: Students do not factorise and expand to move between forms at this stage. The focus is on them recognising the immediately available key features presented by each form.)</td>
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<td><strong>Guided practice:</strong> School-based and online worksheets can be used as resources.</td>
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<td>explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technologies as appropriate (ACMNA239)</td>
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<td>use digital technologies to graph simple quadratics and circles</td>
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<td>recognise and describe equations that represent circles with centre the origin and radius $r$</td>
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<td>sketch circles of the form $x^2 + y^2 = r^2$ where $r$ is the radius of the circle</td>
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<td>establish the equation of the circle with centre $(h, k)$ and radius $r$, and graph equations of the form</td>
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| \((x - h)^2 + (y - k)^2 = r^2\) (Communicating, Reasoning) \(\varnothing\) | Students have applied their understanding of quadratic equations to a 'real-world' application and extended this to begin understanding the equations of circles. **Consolidation for skill development:** The teacher explicitly teaches:  
  - how to sketch circles when the equation given is in the form \(x^2 + y^2 = r^2\) or \((x - h)^2 + (y - k)^2 = r^2\)  
  - (Note to teachers: \(x\)- and \(y\)-intercepts should be identified for circles that have not been translated. For circles that have been translated, students should identify whether the circle intersects with an axis or not, but does not have to calculate the ordinate’s value.)  
  **Guided practice:** School-based and online worksheets could be used as resources. | tml/index.html |
| **Introduction:**  
  - Students confirm their understanding of transformations of the basic parabola explaining the role of each constant in the forms: \(y = ax^2\) and \(y = x^2 + d\)  
  - The teacher show the graph of the basic cubic \(y = x^3\) and challenges students to predict the impact of changes to \(a\) and \(d\) in cubics of the form \(y = ax^3\) and \(y = x^3 + d\)  
  - Students use a digital graphing tool to test their theory | STEM: Begin to develop an understanding of distance/time graphs when an object is moving at a changing velocity.  
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<td><strong>Consolidation for skill development:</strong></td>
<td>The teacher explicitly teaches:</td>
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<td>- how to sketch cubics when the equation is given in the form $y = ax^3$ and $y = x^3 + d$ by identifying orientation, dilation and location from the equation</td>
<td><strong>Technology:</strong></td>
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<td>- $y$-intercept should be identified and labelled</td>
<td>Discuss – What are fonts? Who created them? How does the ‘machine’ know what shapes to construct on screen and communicate to printers?</td>
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<td><strong>Guided practice:</strong> School-based and online worksheets could be used as resources.</td>
<td>Students might be surprised to find that cubic equations are essential to the curves in letters for many fonts.</td>
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<td>- graph and compare features of the graphs of cubic equations of the form $y = a(x - r)(x - s)(x - t)$</td>
<td><strong>Introduction:</strong></td>
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<td></td>
<td>- Students confirm their understanding of reading $x$-intercepts from the factorised form of a quadratic.</td>
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<td>- The teacher presents a fully factorised cubic and challenges students to determine $x$-intercepts.</td>
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<td></td>
<td>- Students use a digital graphing tool to test their theory.</td>
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<td><strong>Consolidation for skill development:</strong></td>
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<td>The teacher explicitly teaches:</td>
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<tr>
<td>- how to sketch cubics when the equation is given in the form $y = a(x - r)(x - s)(x - t)$ by identifying orientation and $x$-intercepts</td>
<td><strong>Note to teachers:</strong> Local maximum and</td>
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<td>- that the $y$-intercept should be found by substituting $x = 0$ into the equation</td>
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<td><em>minimum points are not found at this stage. Students should aim for a ‘smooth curve’ and use the y-intercept as a guide for the path of the curve. It should be pointed out that a cubic does not necessarily have the same symmetry as a parabola.)</em></td>
<td><em>Guided practice:</em> School-based and online worksheets could be used as resources.</td>
<td><em>STEM:</em> Students interested in flight and supersonic flight in particular might like to view this video: ‘Supersonic Flight, Sonic Booms’: <a href="https://www.youtube.com/watch?v=gWGLAAYdbbc&amp;feature=youtu.be">https://www.youtube.com/watch?v=gWGLAAYdbbc&amp;feature=youtu.be</a> which has an explanation of the sonic boom and the visual shock waves created when a plane breaks the sound barrier.</td>
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<td>▪ sketch, compare and describe, with and without the use of digital technologies, the key features of simple exponential curves, e.g. sketch and describe similarities and differences of the graphs of <em>y = 2^x</em>, <em>y = -2^x</em>, <em>y = 2^{-x},</em> <em>y = 2^x + 1, y = 2^x − 1</em></td>
<td><em>Activity:</em> Students consider how low a plane should fly (not for take-off or landing). The term “Hard-Deck” is used by the navy as the lowest altitude a plane should fly at in order to land on the hard deck of the aircraft carries. A “Hard-Deck” could therefore be represented by a dotted line. Students watch a video showing some examples of planes in super-low flight: “Top 10 extremely low flights”: <a href="https://www.youtube.com/watch?v=WOLxNR9dU&amp;feature=youtu.be">https://www.youtube.com/watch?v=WOLxNR9dU&amp;feature=youtu.be</a> On a grid, students sketch a curve showing the height a jet fighter pilot would be above the ground as he or she takes off on a rapid ascent. Students then consider the graph of the equation <em>y = 2^x</em>, and notice the position of the independent variable <em>x</em>. They should comment on the graph, and use their calculator to evaluate: <em>2^{-5}, 2^{-1}, 2^0, 2^1, 2^5, 2^{10}</em> and comment on the <em>y</em>-values and hence the shape of the graph.</td>
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graph when $x < 0$.

The teacher explicitly teaches that graphs of this shape are called exponential graphs (or curves) and all contain a special feature called an asymptote. The asymptote can be thought of as the “Hard-Deck” of the graph.

Students then work in groups using graphing technology to explore the following graphs:

- $y = -2^x$
- $y = 2^{-x}$
- $y = -2^{-x}$
- $y = 2^x + 1$
- $y = 2^x - 1$

Students should comment on what is the same and what is different about each graph and use their reasoning to predict what the graph of $y = -2^x + 3$ might look like, before checking that graph using graphing technology.

**Consolidation for skill development:**

- the teacher guides students to create an appropriate table of values for an exponential curve and practice correct calculator use
- the teacher leads students in practicing the pronunciation of asymptote
- the teacher shows Examples with method for sketching exponential curves

**Guided practice:** School-based and online worksheets could be used as resources, such

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<tr>
<td>▪ graph hyperbolic relationships of the form ( y = \frac{k}{x} ) for integer values of ( k )</td>
<td>Activity: Students complete the worksheet: ‘Dividing by zero”: <a href="https://www.tes.com/teaching-resource/dividing-by-zero-developing-understanding-by-doing-11409592">https://www.tes.com/teaching-resource/dividing-by-zero-developing-understanding-by-doing-11409592</a> in which students are led to understand why dividing by zero returns an error message on their calculators and consider how to graph a hyperbolic relationship from a table of values.</td>
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<tr>
<td>▪ describe, graph and determine the equations of asymptotes for a variety of hyperbolic curves, including where the equation is given in the form ( y = \frac{k}{x} + c ) or ( y = \frac{k}{x-b} ) for integer values of ( k, b ) and ( c ) (Communicating, Problem Solving)</td>
<td>Link to learning: (this is identified in the linked activity)</td>
<td></td>
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<tr>
<td>▪ describe hyperbolas in terms of what happens to the ( y )-values of the points on the hyperbola as ( x ) becomes very large or very small, whether there is a ( y )-value for every ( x )-value, and what occurs near or at ( x = 0 ) (Communicating, Reasoning)</td>
<td>Consolidation for skill development:</td>
<td></td>
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<tr>
<td>▪ determine possible equations from the key features of a graph, eg ( y = 2, y = 2 - x ),</td>
<td>Consolidation of new learning:</td>
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<td>Suggested resources to consolidate and</td>
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<td></td>
<td>STEM: Slow motion videos of parabolas in the real world such as:</td>
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</table>

Consolidation of new learning:

Suggested resources to consolidate and

STEM: Slow motion videos of parabolas in the real world such as:
### Content

\[ y = (x - 2)^2, \quad y = 2^x, \quad (x - 2)^2 + (y - 2)^2 = 4, \]
\[ y = \frac{1}{x-2}, \quad y = 2x^2 \] (Communicating, Reasoning)

- distinguish between positive and negative gradients from a diagram (Reasoning)
- describe and compare a variety of simple non-linear relationships (Communicating, Reasoning)

### Teaching and Learning

extend learning include:

- 'Linear, quadratic and exponential': [Link](https://www.illustrativemathematics.org/content-standards/HSF/LE/clusters)
- Use the worksheet: ‘Finding the gradient of a curve using a tangent’: [Link](https://www.tes.com/teaching-resource/finding-the-gradient-of-a-curve-using-a-tangent-11099940) to extend the concept of gradient beyond positive and negative and introduce students to the understanding of tangents identifying gradient. Consolidate prior learning about gradient of a straight line by calculating the gradient of tangents to a curve.

### STEM Resources and Stimulus

- ‘Snowboard slow motion montage’: [Link](https://www.youtube.com/watch?v=OiXaXRWvqhM&feature=youtu.be). Note the shape of the ramps and flight.
- ‘Tony Hawk Skates First Downward Spiral Loop – BTS’: [Link](https://www.youtube.com/watch?v=gaf8zHP-iaY&feature=youtu.be). Note the perseverance and the way Tony works out where and how he needs to apply force. Note the curves of the different parts of the ramps.
- ‘Slow motion sports HD’: [Link](https://www.youtube.com/watch?v=g0VRiY8COL4&feature=youtu.be). A compilation of many sports. Observe the curves created by people and equipment. Look for straight lines, circles and parabolas. Suggest starting at 1:20
- A Longer video (46 mins) for students interested in Sport Science and Research: ‘Cristiano Ronaldo tested to the limit’: [Link](https://www.youtube.com/watch?v=BeNLD68TTSM&feature=youtu.be)

### Assessment strategies

**Topic Test:** Short-answer and multiple-choice test.

**Student self-evaluation:** Students rate their own development through this unit – their understanding and skills, their application to learning and working mathematically. Students discuss these with one another and then with teacher 'For Learning' in order to identify their readiness to move on to the next topic and to set themselves personal learning objectives they might set themselves for the next topic (participation in class,
Assessment strategies

completion of homework, developing skills).

Resources overview

Teaching & Learning URLs of linked resources:

- ‘Investigate the Parabola 2’: https://www.tes.com/teaching-resource/investigate-the-parabola-2-key-features-11339530
- ‘Investigate the Parabola 3’: https://www.tes.com/teaching-resource/investigate-the-parabola-3-how-the-shape-is-formed-spreadsheet-activity-11437469
- ‘Graphing parabolas in vertex form’: http://www.augustatech.edu/math/molik/GraphParabVertForm.pdf
- ‘Graph quadratics in vertex form’: https://www.khanacademy.org/math/algebra/quadratics/vertex-form-alg1/e/graphing_parabolas_1
- Top 10 extremely low flights”: https://www.youtube.com/watch?v=W0-LXyNR9dU&feature=youtu.be
- ‘Linear, quadratic and exponential’: https://www.illustrativemathematics.org/content-standards/HSF/LE/clusters

STEM Resources & Stimulus URLs of linked resources:

- Video: ‘Hot Robot at SXSW says she wants to destroy humans’: https://www.youtube.com/watch?v=W0_DPi0PmF0&feature=youtu.be
- Video: ‘Are robots today in competition with surgeons?’: https://www.youtube.com/watch?v=MobYs34z5x4&feature=youtu.be
Resources overview

- ‘Snowboard slow motion montage’: https://www.youtube.com/watch?v=OiXaXRWyghM&feature=youtu.be
- ‘Tony Hawk Skates First Downward Spiral Loop – BTS’: https://www.youtube.com/watch?v=gaf8zHp-iaY&feature=youtu.be
- ‘Slow motion sports HD’: https://www.youtube.com/watch?v=g0VRiY8COL4&feature=youtu.be
- ‘Cristiano Ronaldo tested to the limit’: https://www.youtube.com/watch?v=BeNld68TTsM&feature=youtu.be

Sites showing careers that use maths:

- Plus Magazine – career interviews: https://plus.maths.org/content/Career
- Get the Math: http://www.thirteen.org/get-the-math/

Teacher Evaluation of Unit