Mathematics STEM Advanced Pathway

Sample concentrated study unit: Substitute and Solve

The Concentrated Studies focus on skills from within one Strand.

**Duration:** 2 weeks

**Outcomes addressed**

A student:

- provides reasoning to support conclusions that are appropriate to the context (MA5.1-3WM)
- determines the midpoint, gradient and length of an interval, and graphs linear relationships (MA5.1-6NA)
- graphs simple non-linear relationships (MA5.1-7NA)
- sketches and interprets a variety of non-linear relationships (MA5.3-9NA)

**Content**

Students:

- sketch linear graphs using the coordinates of two points (ACMNA215)
- construct tables of values and use coordinates to graph vertical and horizontal lines, such as 
  
  \[ x = 3, x = -1, y = 2, y = -3 \]

- determine whether a point lies on a line by substitution
- complete tables of values to graph simple non-linear relationships, \( y = x^2, y = x^2 + 2, y = 2x \)
- explain what happens to the \( y \)-values of the points on the hyperbola \( y = \frac{k}{x} \) as the \( x \)-values become very large or closer to zero (Communicating)
- explain why it may be useful to choose both small and large numbers when constructing a table of values for a hyperbola (Communicating, Reasoning)
- identify equations whose graph is symmetrical about the \( y \)-axis (Communicating, Reasoning)

**Common misconceptions**

Students may:

- not know how to use calculator functions to ensure order of operations is maintained, eg correct use of brackets
- not understand that their choice of independent values requires consideration if they are to provide useful coordinate pairs for the formation of a curve
- not know how to interpret the calculator error message that dividing by zero will generate
- successfully complete a table of values, but not associate independent and dependent variables in rows as a set of coordinate pairs, let alone points that fall on a curve
- be able to substitute correctly but not rearrange in order to solve for an unknown that is not the subject
- be confused by the large mathematical concepts of asymptotes and infinity
Link to calculus-based courses

The ability to substitute, rearrange and solve is key to solving problems throughout Stage 6 Mathematics.

Developing an in-depth understanding of coordinate pairs as points that fall on (or form) a curve and that these both satisfy and can be calculated from the equation of that curve prepares students for graphing and calculus topics.

Teaching and learning plan

STEM platform - Data

The gathering and utilisation of ‘big data’ is increasing across almost every sector, particularly research and development industries, including those within STEM fields.

Data in itself is of little use, but when data begins to form patterns, those patterns can be used to provide direction for future developments.

When data falls into a recognisable pattern, that pattern can be ‘summarised’ by equations. Those equations then enable forecasting, testing and, significantly, identification of ‘outliers’ and anomalies.

Pre-test: ‘Substitute and Solve’ (TES): https://www.tes.com/teaching-resource/pretest-substitute-and-solve-11437478 which addresses each of the common misconceptions

Introduction activity

(Note to teachers: have graph paper, a spreadsheet and a digital graphing tool ready should students ask for them, but do not suggest use of these until after ‘think, pair, share’.)

Scenario for students

You are in a research lab that is very close to a great breakthrough. Your experiment has produced a large set of data but something keeps going wrong at the last minute. Your research team knows that at least one of the data points must be wrong, maybe more.

You have five minutes to identify the problem pieces of data and remove them so the experiment can run successfully before a rival team takes over your contract.

Here is the data:

(0.22, 10.09) (0.00, error) (7.00, 1.29) (7.30, 1.27) (4.60, 1.43)
(6.40, 1.31) (6.70, 1.30) (1.60, 0.00) (0.43, 5.65) (0.01, 201.00)
(1.50, 2.33) (0.15, 16.33) (8.20, 1.24) (3.40, 1.59) (0.78, 3.56)
(0.57, 4.51) (2.20, 1.91) (0.50, 5.00) (7.60, 1.26) (5.20, 1.38)

Students ‘think, pair, share’ a method of solving this problem and then apply their chosen method, working against the clock.
**Link to learning**

Students may have:

- assumed the data points (0.00, error) and (0.01, 201.00) were faulty, based on their ‘appearing different’
  - this can lead to discussion of valid ‘error’ results for some calculations and valid ‘extreme’ results for some relationships
- entered data points into a spreadsheet and sorted them to reveal whether any y-values fell out of sequence
  - utilising the versatility and efficiency of this tool
- graphed the data points to see if any stood out from a pattern created by the majority
  - utilising a visual pattern’s ability to identify anomalies

**Activity: Does this belong?**

- The activity: ‘Which one doesn’t belong?’ is located at: [http://wodb.ca/index.html](http://wodb.ca/index.html).
- As described in the activity, the solutions are not provided and multiple correct answers are possible. The challenge is to be able to justify the obtained answer.

**Link to learning**

Developing student capacity to look critically at a set of information and determine similarities and differences is beneficial to many mathematical processes. This is further enhanced by asking students to articulate their thinking and decision-making. In the learning that follows, students will be asked to check their answers for anomalies and hence identify potential errors in their workings.

Points that satisfy an equation or fall on a line ‘belong’ to the relationship described by that equation and graphed by that line.

**Consolidation for skill development**

1. Testing points – graphical method
   - Recall Stage 4 Linear Relationships: creating a table of values and plotting points to form a line.
   - Worksheet: ‘Does this point belong?’ (TES): [https://www.tes.com/teaching-resource/does-this-point-belong-11401546](https://www.tes.com/teaching-resource/does-this-point-belong-11401546). Discussion after completion of the worksheet – How many points do you need to plot to be able to judge whether one does not belong with the rest?
   - Plotting curves by creating tables of values – hence testing calculations by producing a smooth curve
     (Note to teachers: While students will produce curves of linear and quadratic equations here, the intention of the task is for them to practise multiple ‘substitute and solve’ calculations. For this reason, a calculator and not a spreadsheet should be used throughout this short sequence. The fact that students generate a curve will be used to allow students to identify any calculation errors. Some students may be able to see the pattern forming in their y-values even without plotting points.)

   Ensure students can correctly use the appropriate functions of their calculators in order to complete tables of values for each type of function:

   - Create worksheets on ‘Graphing from Function Tables’ at: [https://www.worksheetworks.com/math/geometry/graphing/function-table-graph.html](https://www.worksheetworks.com/math/geometry/graphing/function-table-graph.html)
- Watch the video ‘How do you make a table of values for a linear equation’ located at (Virtual Nerd): http://www.virtualnerd.com/pre-algebra/linear-functions-graphing/equations/introduction-linear-equations/generate-table-values-example

Content:
- Table of values for horizontal and vertical lines.
- Discuss: Does \( y = 7 \) look like an equation or the answer to an equation? [class vote]; What about \( x = -2 \)? [class vote]
- Table of values for exponential and hyperbolic functions
  (Note to teachers: These functions will be studied fully in later graphing topics. The emphasis here is on correct calculator use in order to generate a table of values.)
  - Discuss \( y \)-values in hyperbolic relationships as \( x \)-values get closer to zero.
  - What does ‘close to zero’ mean?
  - How close can their calculators get to zero and still obtain an answer?
  - What does infinity and negative infinity mean?
  - What does ‘not defined’ mean and how is it different to zero?

2. Testing points – algebraic method
- Watch the video illustrating the structured method: ‘How do you determine if an ordered pair is a solution to an equation’ (Virtual Nerd): http://www.virtualnerd.com/pre-algebra/algebra-tools/relations-ordered-pairs-coordinate-plane/ordered-pairs-coordinate-plane/test-ordered-pair-equation-solution

Guided practice

School-based and online resources
- Extension: ‘How to recognise the type of graph from a table’: http://pdfs.cpm.org/state_supplements/Linear_Quadratic_Exponential_Tables.pdf
- Structured: Use the interactive page: ‘Determine if each a point lies on a given line’: http://flexmath.ck12.org/lesson/algebra/determine-if-a-point-lies-on-a-given-line/presentation A linear equation is given. Students use graphical and algebraic methods to test points.

Resources

URLs for linked items
The following URLs are correct at the time of publication
- "Which one doesn’t belong?": http://wodb.ca/index.html
- Does this point belong? (TES): https://www.tes.com/teaching-resource/does-this-point-
belong-11401546

- ‘Special cases for tables of values – Part 1’ (TES): https://www.tes.com/teaching-resource/special-cases-for-tables-of-values-part-1-11401553
- ‘Special cases for tables of values – Part 2’ (TES): https://www.tes.com/teaching-resource/special-cases-for-tables-of-values-part-2-11401554
- ‘How do you determine if an ordered pair is a solution to an equation?’ (Virtual Nerd): http://www.virtualnerd.com/pre-algebra/algebra-tools/relations-ordered-pairs-coordinate-plane/ordered-pairs-coordinate-plane/test-ordered-pair-equation-solution
- ‘How to recognise the type of graph from a table’: http://pdfs.cpm.org/state_supplements/Linear_Quadratic_Exponential_Tables.pdf
- ‘Determine if each a point lies on a given line’ (cK-12 Engaging, interactive lessons): http://flexmath.ck12.org/lesson/algebra/determine-if-a-point-lies-on-a-given-line/presentation