Stage 5: Mathematics STEM Advanced Pathway

Sample concentrated study unit: Rearrange and Solve (sample programming proforma with some direction)

The Concentrated Studies focus on skills from within one Strand.

**Duration:** 3 weeks

**Outcomes**

A student:

- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems (MA5.2-2WM)
- solves complex linear, quadratic, simple cubic and simultaneous equations, and rearranges literal equations (MA5.3-7NA)
- solves financial problems involving compound interest (MA5.2-4NA)

**Content**

Students:

- solve complex linear equations involving algebraic fractions
- solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)
- solve a variety of quadratic equations eg $3x^2 = 4$, $x^2 - 8x - 4 = 0$, $x(x - 4) = 4$, $(y - 2)^2 = 9$
- choose the most appropriate method to solve a particular quadratic equation (Problem Solving)
- check the solutions of quadratic equations by substituting
- solve quadratic equations resulting from substitution into formulae
- substitute a pronumeral to simplify higher-order equations so that they can be seen to belong to general categories and then solve the equations, eg substitute $u$ for $x^2$ to solve $x^4 - 13x^2 + 36 = 0$ for $x$
- solve simple cubic equations
- determine that for any value of $k$ there is a unique value of $x$ that solves a simple cubic equation of the form $ax^3 = k$ where $a \neq 0$
- explain why cubic equations of the form $ax^3 = k$ where $a \neq 0$ have a unique solution (Communicating, Reasoning)
- solve simple cubic equations of the form $ax^3 = k$, leaving answers in exact form and as decimal approximations
- rearrange literal equations
- change the subject of formulae, including examples from other strands and other learning areas, eg make $a$ the subject of $v = u + at$, make $r$ the subject of $\frac{1}{x} = \frac{1}{r} + \frac{1}{s}$, make $b$ the subject of $x = \sqrt{b^2 - 4ac}$
- determine restrictions on the values of variables implicit in the original formula and after rearrangement of the formula, eg consider what restrictions there would be on the variables in the equation $Z = ax^2$ and what additional restrictions are assumed if the equation is rearranged to $x = \frac{\sqrt{Z}}{a}$ (Communicating, Reasoning)
- solve simultaneous equations, where one equation is non-linear, using algebraic and graphical techniques, including the use of digital technologies
use analytical methods to solve a variety of simultaneous equations, where one equation is non-linear, eg \( \begin{cases} y = x^2 \\ y = x \end{cases}, \begin{cases} y = x^2 - x - 2 \\ y = x + 6 \end{cases}, \begin{cases} y = x + 5 \\ y = \frac{6}{x} \end{cases} \)

- choose an appropriate method to solve a pair of simultaneous equations (Problem Solving, Reasoning)
- solve pairs of simultaneous equations, where one equation is non-linear, by finding the point or points of intersection of their graphs using digital technologies
- determine and explain that some pairs of simultaneous equations, where one equation is non-linear, may have no real solutions (Communicating, Reasoning)
- determine the points of intersection of a line with a parabola, hyperbola or circle, graphically and algebraically

**Differentiation:**

- *Extension* - calculate the principal or interest rate needed to obtain a particular total amount for a compound interest investment (Problem Solving)
- *Extension* - use a 'guess and refine' strategy to determine the number of time periods required to obtain a particular total amount for a compound interest investment (Problem Solving)

**Common misconceptions**

Students may:

- be ‘process-based’ about order of operations and need to begin seeing equations in their entirety in order to become more fluid when rearranging them
- be confusing subtraction (\(-\)) with negative (-)
- be confronted by equations involving algebraic fractions
- be yet to develop their understanding of equations as describing how variables interact
- be yet to recognise equations as describing a set of points that form a curve and hence all points on a curve satisfy the equation
- be yet to develop the habit of checking their solutions against their general knowledge in order to evaluate their answers

**Link to calculus-based courses**

The ability to select an appropriate tool (formula or algorithm), rearrange it to suit a problem and substitute given information correctly in order to solve for an unknown is key to Stage 6 Mathematics courses.

**Teaching and Learning plan**

**STEM platform – Problem solving**

The skills within this concentrated study are introduced and practised in other Mathematics STEM Advanced Pathway ‘topics’. Students who have been following the recommended Scope and Sequence will have seen real-world applications of these skills.

For this reason, the STEM platform for ‘Rearrange and Solve’ is problem solving, or more specifically, challenging students to ‘go to their toolbox’ of mathematical skills and select an appropriate combination of ‘tools’ to address open-ended or un-scaffolded questions.

Focus question: ‘What do I do when I don’t know what to do?’

**Pretest**
Three questions only:

1. Solve $x^4 - 13x^2 = -36$
2. Do the circle with equation $(x - 2)^2 + (y - 2)^2 = 4$ and the parabola with equation $2y = x^2 - 4x + 4$ intersect? Give mathematical evidence for your answer.
3. Extension: You want to invest $5000. Two products are available. One pays simple interest at 4.5%, the other pays compound interest at 3%. Describe the factors you need to consider when choosing between these loans and give an example of the mathematics you would use to inform your final decision.

Introduction activity

In preparation for the first lesson students gather all the formulae they use in other subjects from their own textbooks, workbooks and teachers and present these in posters for display in the classroom. Variables should be defined and units included where they exist. These can be referred to as the learning sequence progresses.

Link to learning

Increase student awareness of how often they use formulae and how often they need to rearrange a formula to suit a particular problem.

Discussion: In other subjects, must students select formulae themselves or are they told which one to use within a question? Do they memorise those formulae or are formulae provided in test situations?

Consolidation for skill development

- Depending on class/student readiness, rather than explicit teaching of Content, this concentrated study can be presented as a series of problems for students to solve by combining skills from previous topics.
- In particular, students will need to develop confidence with algebraic substitution and begin to see this as a useful tool allowing manipulation of equations.
- Compare graphical and algebraic solutions where appropriate and use one to confirm the other.

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Resources

Teaching and Learning URLs for linked items

- ‘Because I can do these – I can work out how to do this’ (TES): [https://www.tes.com/teaching-resource/because-i-can-do-these-i-can-work-out-how-to-do-this-developing-past-learning-into-new-skills-11451637](https://www.tes.com/teaching-resource/because-i-can-do-these-i-can-work-out-how-to-do-this-developing-past-learning-into-new-skills-11451637)
- ‘Applications of Quadratic Functions’ (Money Institute): [http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html](http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html)